

# Nonparametric Sparse Learning of Nonlinear Dynamical Systems

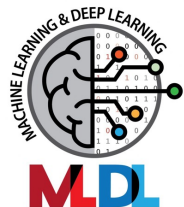
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Sandia National Laboratories Annual ML/DL Workshop, 2024

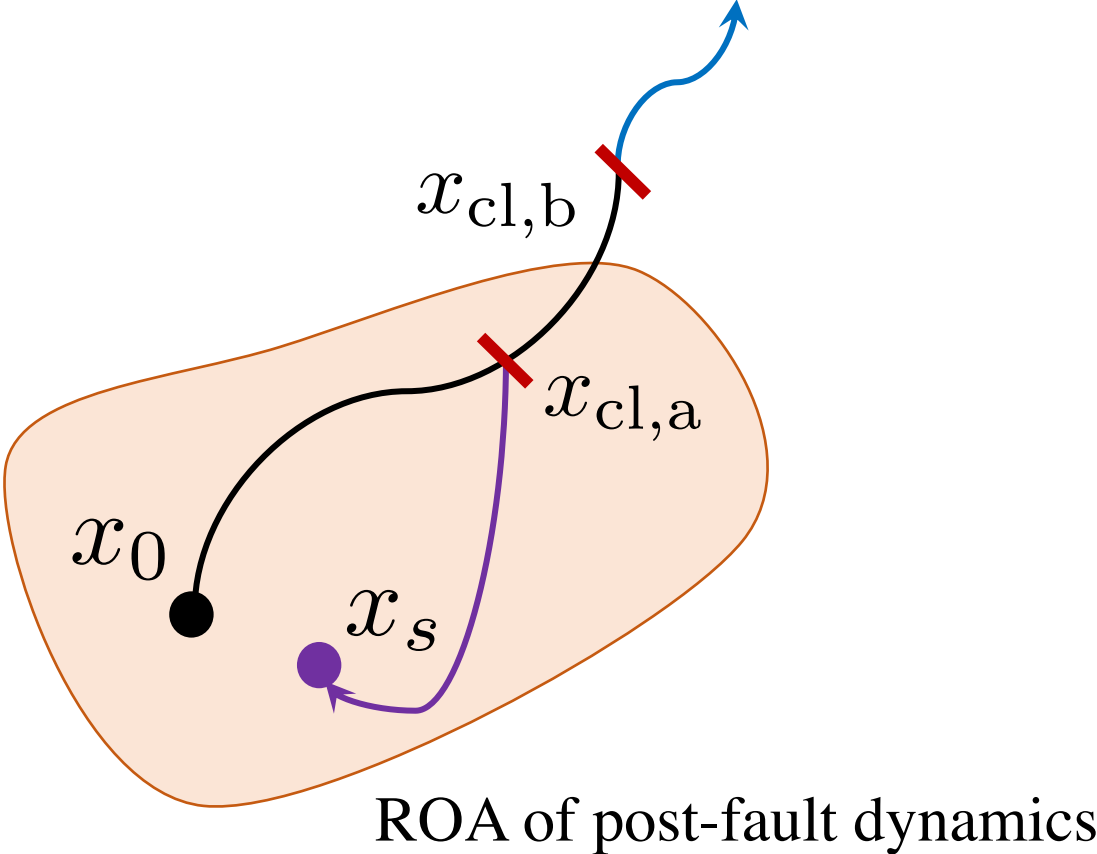


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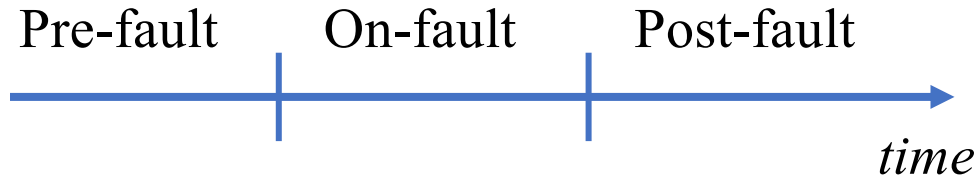


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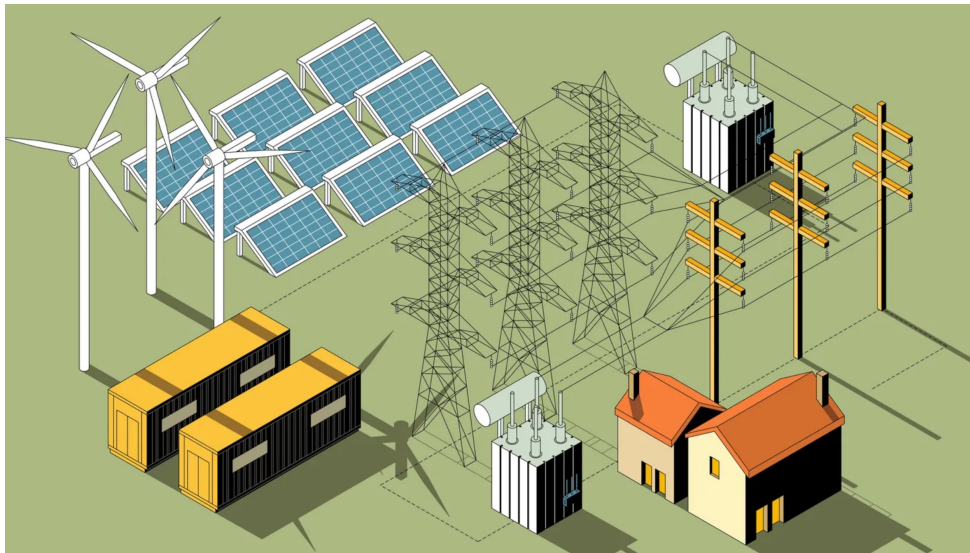
# Data Driven Transient Stability Analysis



# Data Driven Transient Stability Analysis via Koopman Operator



- Numerical integration / energy functions
- From data?



Markovian dynamical system

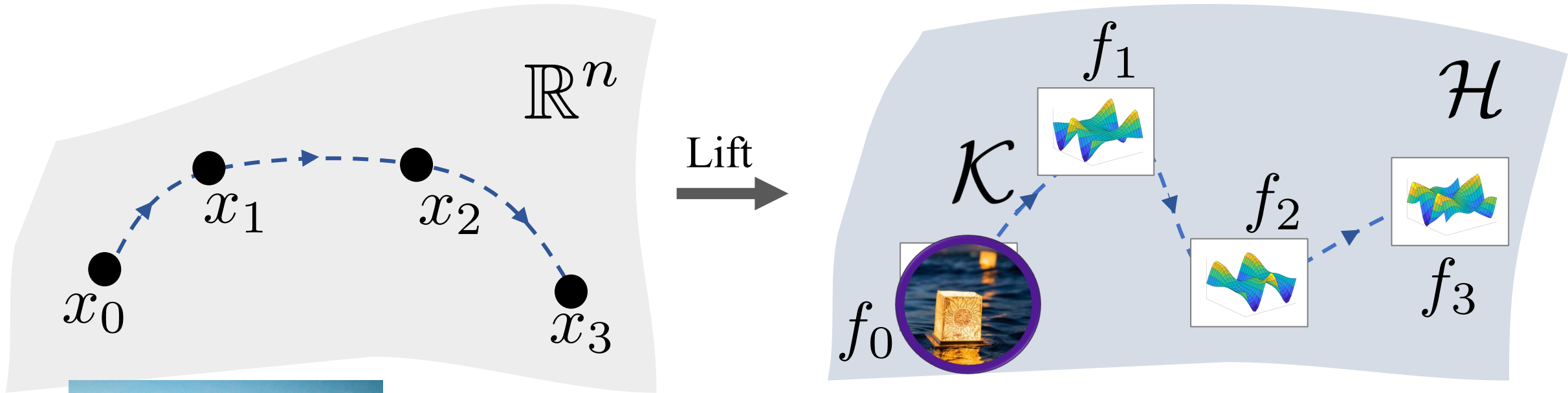
$$\mathbb{P}\{x_{t+1} \in \mathbb{A} | x_t = x\} = \int_{\mathbb{A}} p(y|x) dy,$$

Koopman operator

$$(\mathcal{K}f)(x) = \int p(y|x) f(y) dy.$$

$\mathcal{K}$  as a representation of system dynamics

Koopman Operator  $(\mathcal{K}f)(x) = \mathbb{E}[f(X_{t+1}) | X_t = x]$  [Koopman 31]

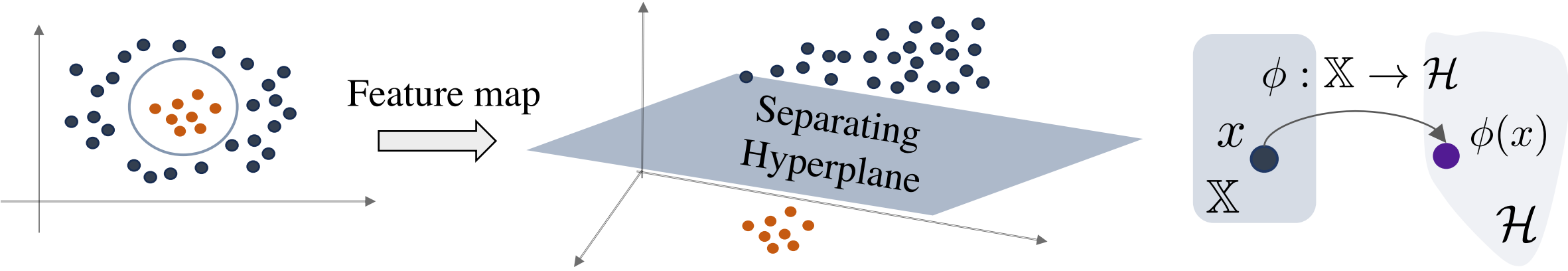


Given  $\{x_0, \dots, x_m\}$ , learn  $\hat{K}$  s.t.

$$\|\hat{K} - \mathcal{K}\|_{\text{op}} \leq \varepsilon \quad \text{w.h.p.}$$

in the reproducing kernel Hilbert space

# Koopman Operators on Reproducing Kernel Hilbert Space

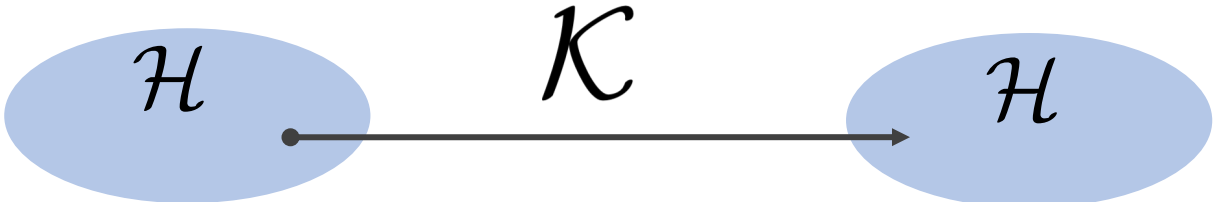


Consider a Reproducing Kernel Hilbert Space (RKHS)  $\mathcal{H}$  with kernel  $\kappa(\cdot, \cdot)$

Canonical feature map:  $\phi(x) = \kappa(\cdot, x)$

Denote the covariance operators as  $C_{XX^+} := \mathbb{E}[\phi(X) \otimes \phi(X^+)]$  ,  $C_{XX} := \mathbb{E}[\phi(X) \otimes \phi(X)]$

When interacting with RKHS, the Koopman operator can be written as  $\mathcal{K} = C_{XX}^\dagger C_{XX^+}$  [Klus '20]



# Nonparametric **Sparse** Learning of Nonlinear Dynamical Systems

The regularized Koopman operator  $\mathcal{K}_\lambda = (C_{XX} + \lambda I)^{-1} C_{XX+}$

Given a dataset  $D = \{(x_1, x_1^+), \dots, (x_M, x_M^+)\}$

**Model complexity is captured by  $|D| = M$**

Empirical estimates of covariance operators

$$C_{X^+X}^M = \frac{1}{M} \sum_{i=1}^M \phi(x_i^+) \otimes \phi(x_i), \quad C_{XX}^M = \frac{1}{M} \sum_{i=1}^M \phi(x_i) \otimes \phi(x_i).$$

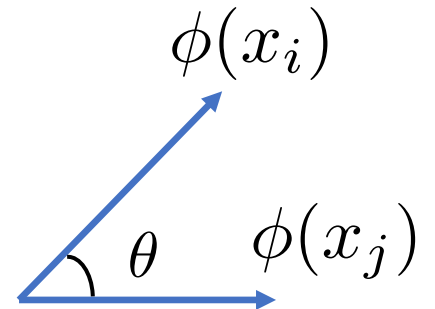
The regularized empirical estimate are  $K_\lambda^M = (C_{XX}^M + \lambda I)^{-1} C_{X^+X}^M$

Prune  $D$  to construct a sparse dictionary  $D_\gamma \subseteq D$  via *the coherence condition*

$$\frac{|\langle \phi(x_i), \phi(x_j) \rangle_{\mathcal{H}}|}{\|\phi(x_i)\|_{\mathcal{H}} \|\phi(x_j)\|_{\mathcal{H}}} \times \frac{|\langle \phi(x_i^+), \phi(x_j^+) \rangle_{\mathcal{H}}|}{\|\phi(x_i^+)\|_{\mathcal{H}} \|\phi(x_j^+)\|_{\mathcal{H}}} \leq \gamma, \forall (x_i, x_i^+), (x_j, x_j^+) \in D_\gamma$$

Sparse kernel Koopman operator  $\hat{K}_\lambda = (\hat{C}_{XX} + \lambda I)^{-1} \hat{C}_{X^+X}$

**Model complexity is captured by  $|D_\gamma| \ll |D|$**



# Nonparametric **Sparse** Learning of Nonlinear Dynamical Systems

Sparse Koopman operator  $\hat{K}_\lambda = (\hat{C}_{XX} + \lambda I)^{-1} \hat{C}_{XX+}$

Leading eigenfunction

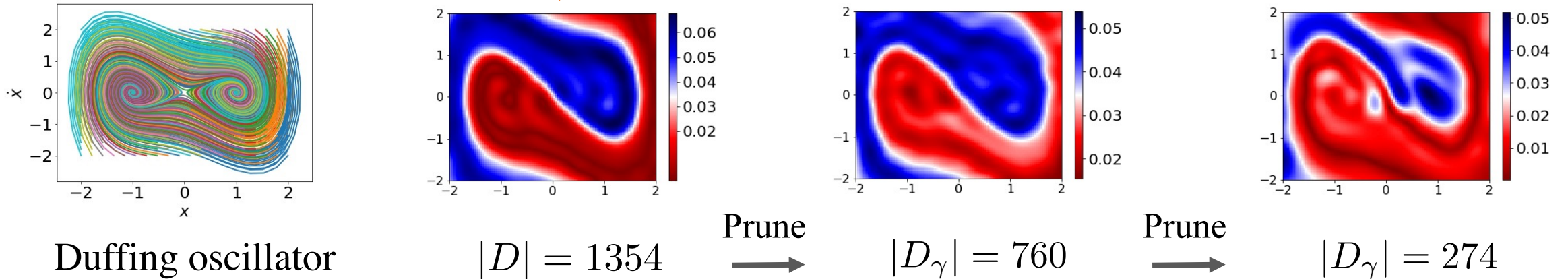


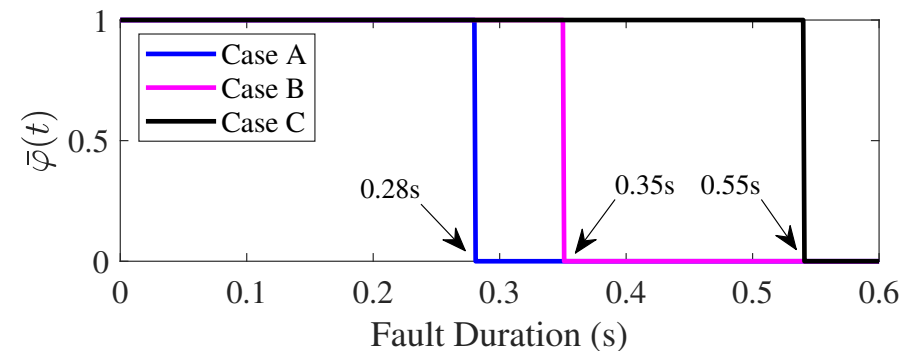
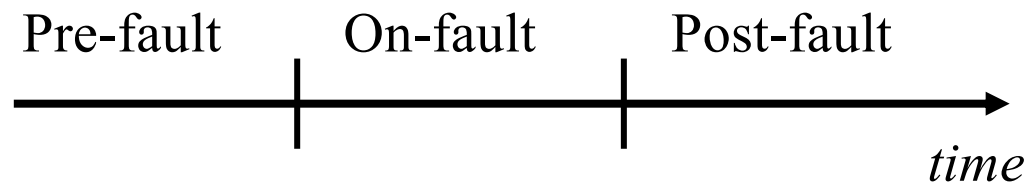
Figure: Estimated regions of attraction with sparse dataset  $D_\gamma \subseteq D$ . [Hou et al. 21]

An example of theoretical guarantees: high probability bound with IID samples. [Hou et al ICML 23]

$$\|\mathcal{K}_\lambda - \hat{K}_\lambda\|_{\text{op}} \leq [\text{sampling error}(\frac{1}{\sqrt{M}}) + \text{sparsification error}(\sqrt{1 - \gamma^2})] \mathcal{O}(\frac{1}{\lambda^2})$$

- IID samples  $\rightarrow$  Trajectory-based sampling. [Hou et al ICML 23]
- Batch learning  $\rightarrow$  Learning from streaming samples with last-iterate convergence guarantees. [Hou et al 24b]
- A similar framework for learning generators of diffusion processes. [Hou et al ICML 23]

# Koopman Operator for Transient Stability Analysis of Power Systems

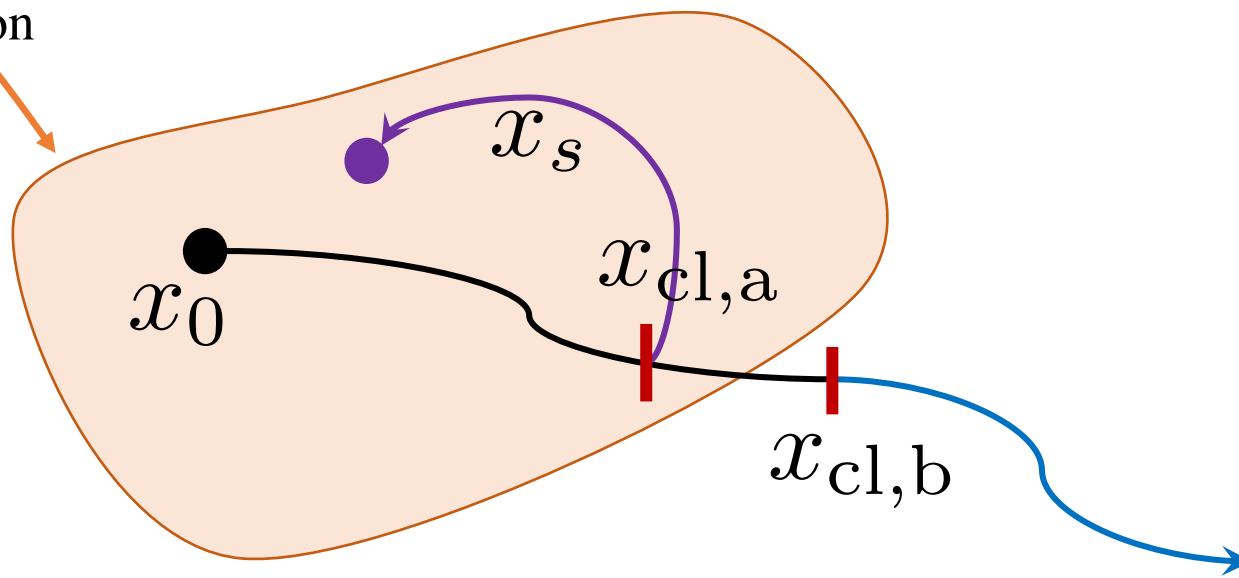


Sparse kernel Koopman operator

$$\hat{K}_\lambda = (\hat{C}_{XX} + \lambda I)^{-1} \hat{C}_{XX+}$$

Leading eigenfunction

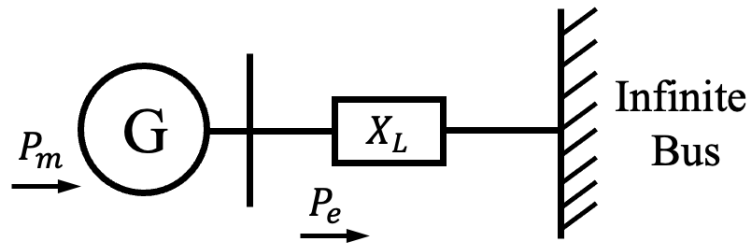
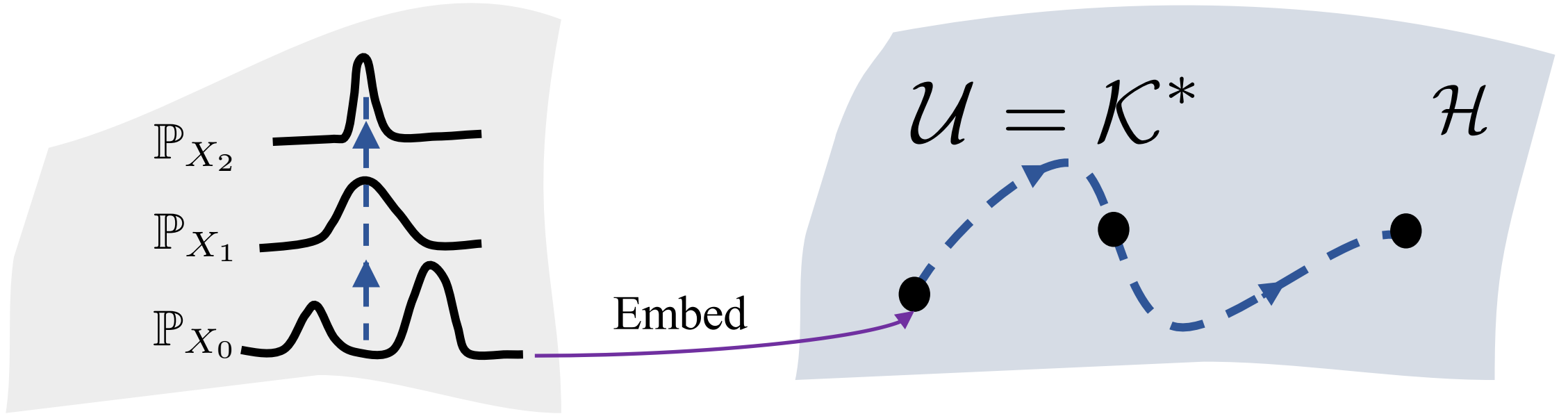
ROA of post-fault dynamics



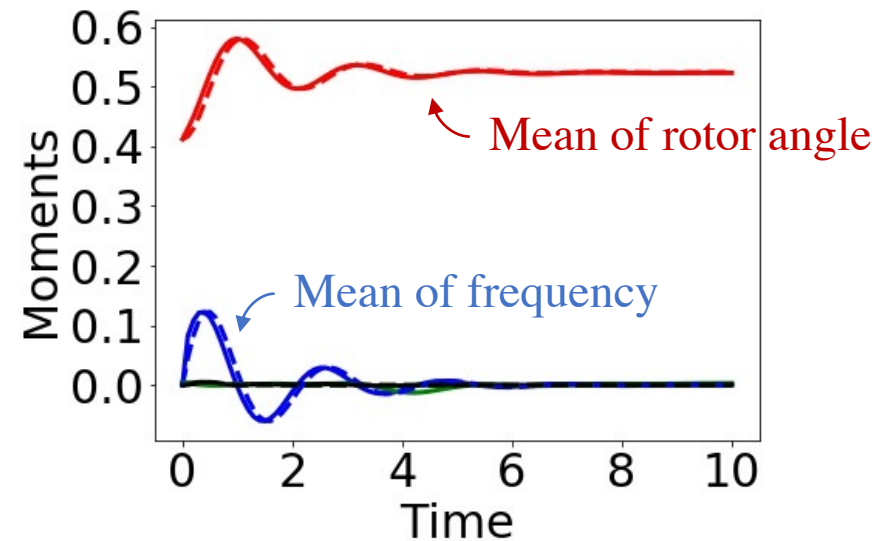
IEEE 39 bus system: Evaluate the leading eigenfunction of  $K$  along on-fault trajectories to estimate critical clearing time. [Hou et al. 24a]



# Propagating Uncertainty in Initial Conditions [Hou et al. Physica D 24]



Single Machine Infinite Bus Systems:  
uncertainties in the initial conditions are  
rotor angles of the generator



# Summary of Contributions

Non-parametric identification of nonlinear systems with controlled growth  
in model complexity and finite-sample performance guarantees.

Acknowledgement: S. Bose (UIUC), U. Vaidya (Clemson), N. Dahlin (SUNY), S. Sanjari (RMC), H. Choi (SNL),  
A. Matavalam(ASU), M. Raginsky (UIUC), R. Srikant (UIUC), T. Basar (UIUC)

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[Hou et al. AAAI 23] Hou, Boya, Sina Sanjari, Nathan Dahlin, and Subhonmesh Bose. Compressed decentralized learning of conditional mean embedding operators in Reproducing Kernel Hilbert Space. In Proceedings of the 37th AAAI Conference on Artificial Intelligence (AAAI), 2023.

[Hou et al. Physica D 24] Hou, Boya, Amarsagar Reddy Ramapuram Matavalam, Subhonmesh Bose, and Umesh Vaidya. Propagating uncertainty through system dynamics in reproducing kernel Hilbert space. Physica D: Nonlinear Phenomena, 463:134168, 2024.

[Hou et al. 24a] Amarsagar Reddy Ramapuram Matavalam, Hou, Boya, Hyungjin Choi, Subhonmesh Bose, and Umesh Vaidya. Data-driven transient stability analysis using the Koopman operator. International Journal of Electrical Power and Energy Systems (Under revision), 2024.

[Hou et al. 24b] Boya Hou, Sina Sanjari, Alec Koppel, and Subhonmesh Bose. Compressed online learning of conditional mean embedding. arXiv preprint arXiv:2405.07432, 2024



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# Thank you !

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